REGULAR HEAT TRANSFER OF EXTERNAL WALLS

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A method of isolating the regular solution from the numerical solution is proposed. The accuracy of solution according to explicit and implicit schemes is considered, and the applicability conditions are determined.

Consider the nonsteady heat conduction in multilayer walls with boundary conditions of the third kind

$$c(x)\gamma(x)\frac{\partial u}{\partial x}=\frac{\partial}{\partial x}\left(\lambda(x)\frac{\partial u}{\partial x}\right), \tag{1}$$

$$\lambda(x)\frac{\partial u}{\partial x}\bigg|_{x=0}=\beta_1 u(0,\ t)-\mu_1, \tag{2}$$

$$-\lambda(x)\frac{\partial u}{\partial x}\bigg|_{x=l}=\beta_2 u(l,\ t)-\mu_2, \tag{3}$$

$$\beta_1=\alpha_{\mathrm{E}},\ \beta_2=\alpha_{\text{в}},\ \mu_1=\alpha_{\mathrm{E}}u_{\mathrm{E}}(t),$$
$$\mu_2=\alpha_{\mathrm{I}}u_{\mathrm{I}}(t),\ u(x,\ 0)=u_0(x).$$

The necessity of solving Eqs. (1)-(3) arises in the period of extremal winter and summer depredations of the external environment; for example, the temperature variation in the period of sharp cooling is represented by a parabolic trend with the superposition of random harmonic oscillations. The known analytical solutions [1-6] take no account of arbitrary initial conditions, since these conditions, generally speaking, are undetermined. Specifying uniform initial conditions [5, 6] leads to a solution consisting of two parts: in the first, account is taken of the initial conditions, whose influence dies away exponentially with time; and the second is the regular part of the solution, independent of the initial conditions. In [7], a semiempirical model was constructed for calculating the internal air temperature of buildings in winter; the calculated points were described as "calculation with incorrectly specified initial conditions," but at $t > 26$ h the numerical solution obtained practically coincides with actual observations, i.e., regular heat transfer uninfluenced by the initial conditions begins. Hence, in using any method to solve the system of heat- and mass-transfer equations for a building of the type in [7, 8], the calculations must be performed in the region of regular heat transfer.

For walls with different D

$$D=\sum_{i=1}^{N} l_i\,(\omega/a_i)^{1/2}=\sum_{i=1}^{N} Pd_i^{1/2}$$

τ_{reg} is different; the possibility of using D as an approximate integral characteristic of the thermophysical properties follows from analytical solutions for regular temperature waves [1-6]. In connection with this, one further need to calculate τ_{reg} should be noted: representation of the external temperature in Fourier-series form is expedient both in the analysis of numerical calculations and in obtaining analytical solutions, but prolongation of the intervals T is associated with rise in the number of terms in the series. Thus, when using meteorological data with an observation interval of 6 h, there are ten terms in the series when T = 120 h, but only two when T = 24 h. Therefore, for walls with different D, in calculations of the nonsteady heat transfer, it is expedient to use time series for the external-air temperature of different lengths.

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In view of the linearity of Eq. (1) and the boundary conditions in Eqs. (2) and (3), the heat transfer with u_I = const may be investigated, and u_E may be specified by only one harmonic of the Fourier-series expansion of the external-air temperature

$$u_E = u_c + A_0 \cos \omega t. \tag{4}$$

This linear problem may be regarded as consisting of two: one with arbitrarily specified initial distribution of the temperature u_0 and with u_I = const, $u_E = u_c$; and the other with zero initial distribution, $u_0 = 0$, zero temperature of the internal medium, and external temperature varying according to the law

$$u_E = A_0 \cos \omega t. \tag{5}$$

As calculations show, τ_{reg} for the second problem is larger than τ_{reg} for the first when linear initial distributions varying over broad limits are specified, corresponding to the possible combinations of external and internal temperatures of the medium typical of external walls. Therefore, τ_{reg} is investigated for Eq. (1) with the conditions

$$\lambda \frac{\partial u}{\partial x}\bigg|_{x=0} = \alpha_E (u - A_0 \cos \omega t), \quad -\lambda \frac{\partial u}{\partial x}\bigg|_{x=l} = -\alpha_I u, \quad u(x, 0) = 0. \tag{6}$$

The solution for the internal-surface temperature in the region of regular heat transfer is written in the form [9]

$$u = A \cos \omega t + B \sin \omega t. \tag{7}$$

A procedure for calculating τ_{reg} in the case of numerical solution of Eq. (1) with the conditions in Eq. (6) has been developed. Suppose that, for times $t_i \ldots t_{i+j} \ldots t_{i+k-1}$, the temperature values at the internal surface of the wall are found. To determine A and B by the method of least squares, these k points are approximated by a curve of the form in Eq. (7). At each point i + j, it is required that $|(u - u_{i+j})u_{i+j}| \leqslant \varepsilon$; when this condition is satisfied for all i + j, the τ_{reg} corresponding to t_i is taken as the time of onset of regular heat transfer at the internal surface of the wall. In the calculations, the remote points were subject to approximation. For example, with a time step $\tau \approx 0.001$ h, points 1 h apart were chosen, and k was in all cases set equal to ten. Thus, in each hour, the last ten temperature values were approximated by the curve in Eq. (7). Specifying the relative condition of convergence ensures that τ_{reg} will be independent of the value of A_0 in Eq. (5).

Equation (1) is approximated by the difference equation [10]

$$A_i \hat{y}_{i-1} - C_i \hat{y}_i + B_i \hat{y}_{i+1} = y_i, \quad i = 1, \ldots, n.$$
$$A_i = \frac{\lambda_i \tau}{h_i \hbar_i}, \quad B_i = \frac{\lambda_{i+1}\tau}{h_{i+1}\hbar_i}, \quad C_i = A_i + B_i + 1, \tag{8}$$
$$\hbar_i = 0.25 (h_i + h_{i+1})(c_i \gamma_i + c_{i+1}\gamma_{i+1}), \quad \hat{y} = y(x, t + \tau).$$

This difference equation has an accuracy $O(\tau + h^2)$; the boundary conditions of the third kind are also approximated with an accuracy $O(\tau + h^2)$.

$$\hat{y}_0 = \varkappa_1 \hat{y}_1 + \nu_1, \quad \hat{y}_n = \varkappa_2 \hat{y}_{n-1} + \nu_2,$$
$$\varkappa_1 = \frac{2\lambda_1 \tau}{(\lambda_1 + h_1\beta_1) 2\tau + h_1^2 c_1 \gamma_1}, \quad \varkappa_2 = \frac{2\lambda_n \tau}{(\lambda_n + h_n \beta_2) 2\tau + h_n^2 c_n \gamma_n}, \tag{9}$$
$$\nu_1 = \frac{h_1^2 c_1 \gamma_1 y_0 + 2\tau \bar{\mu}_1 h_1}{(\lambda_1 + h_1 \beta_1) 2\tau + h_1^2 c_1 \gamma_1}, \quad \nu_2 = \frac{h_n^2 c_n \gamma_n y_n + 2\tau \bar{\mu}_2 h_n}{(\lambda_n + h_n \beta_2) 2\tau + h_n^2 c_n \gamma_n}.$$

The system in Eq. (8) is solved by the trial-and-error method. Since the calculation time with respect to t is no less than τ_{reg}, there arises the question of the choice of time step. Although the implicit scheme is absolutely stable, a constraint must be imposed on τ, when $t \approx \tau_{reg}$, to increase the accuracy, i.e., the convergence of the numerical solution when $t > \tau_{reg}$ to the solution corresponding to regular heat transfer. The constraints on the step τ to meet this requirement are approximately the same as the constraints on the step in the explicit scheme.

Table 1 gives the results of calculations for periods T corresponding to Fourier-series expansion of the time series of the temperature of length 120 h. The results are compared with respect to the nonstationarity coefficient $\psi = A_t/\alpha_I R$ [8]; from Eq. (7), $A_t = (A^2 + B^2)^{1/2}$.

TABLE 1. Values of ψ for a Wall with D = 1.21, Bi_E = 5.05, Bi_I = 1.08

T	Accurate solution	Explicit scheme	Implicit scheme		
		$\tau=\tau_m/10$	$\tau=\tau_m$	$\tau=1$ h	$\tau=6$ h
120	0,9981	0,9982	0,9992	0,9955	0,9828
60	0,9923	0,9928	0,9969	0,9821	0,9359
40	0,9829	0,9840	0,9931	0,9606	0,8705
30	0,9701	0,9720	0,9879	0,9322	0,7988
24	0,9542	0,9570	0,9812	0,8980	0,7308
20	0,9356	0,9394	0,9732	0,8596	0,6722
17,1	0,9145	0,9195	0,9639	0,8183	0,6260
15	0,8916	0,8978	0,9534	0,7754	0,5930
13,3	0,8712	0,8739	0,9419	0,7320	0,5733
12	0,8414	0,8499	0,9294	0,6888	0,2334

The minimum time step for a four-layer wall (Table 1) is 0.0106 h; when τ = 1 h, the nonstationarity coefficient for T = 12 h is 30% less than that found for accurate solution [4]; for example, the step τ = 1 h is taken in [9]; for τ = 6 h, the corresponding reduction reaches 70%. Thus, the results of the calculations show that using arbitrary time steps in calculations by the implicit scheme may lead to considerable distortion of the temperature values on the internal surface of the wall. Decrease in the time step increases the time required for the calculations, and hence the explicit scheme would be expected to be more economical for the determination of τ_{reg}.

The difference equations for calculations by the explicit scheme [10] are

$$\hat{y}_i = (1 - A_i - B_i)\, y_i + A_i y_{i-1} + B_i y_{i+1}, \tag{10}$$

and, omitting the computations, the difference approximation of the boundary conditions of the third kind may be written in the form

$$\hat{y}_0 = y_0 + (y_1 - y_0)\frac{2\lambda_1\tau}{h_1^2 c_1\gamma_1} - (\beta_1 y_0 - \overline{\mu}_1)\frac{2\tau}{h_1 c_1\gamma_1},$$

$$\hat{y}_n = y_n - (y_n - y_{n-1})\frac{2\lambda_n\tau}{h_n^2 c_n\gamma_n} - (\beta_2 y_n - \overline{\mu}_2)\frac{2\tau}{h_n c_n\gamma_n}.$$

This scheme is conditionally stable: $\tau < \tau_m$, the condition for asymptotic stability, coincides with the stability condition; calculations by Eq. (10) are shown in Table 1.

The accuracy of calculations with $\tau = \tau_m$ in the explicit scheme is higher than for the implicit scheme with τ = 1 and 6 h; increase in accuracy is achieved with reduction in the step τ. Thus, at $\tau = \tau_m/10$, Δ for ψ of higher harmonics is improved; when $\varepsilon = 10^{-5}$ the error is $\approx 5\cdot 10^{-3}$, which corresponds to an error of the temperature at the internal surface of the wall of $\approx 5\cdot 10^{-4}$°C; at low harmonics, the error ($\sim 10^{-4}$) corresponds to an error in the temperature of $\sim 10^{-5}$°C. Increase in accuracy of the higher harmonics is achieved by further reduction in τ. At $\tau = \tau_m/20$, ψ differs from the accurate value by $3.6\cdot 10^{-3}$, which is three times less than at $\tau = \tau_m/10$; in this case, τ_{reg} rises by 2 h, and hence the error in determining τ_{reg} depends on the order of accuracy of the scheme and on the method of isolating the regular solution from the numerical solution. Thus, further improvement in accuracy with respect to the correct solution requires change in the grid parameters, and selection of a more complex method of isolating the regular solution, which inevitably entails an increase in the computer time required. However, the adopted grid parameters and $\varepsilon = 10^{-5}$ provide good agreement of the numerical and accurate solutions, while graphical means offer the possibility of determining the limit of the region of regular heat transfer approximately, and show that, for the calculation of nonsteady heat transfer and hence for the choice of calculation temperatures for [11, 12], the duration of the sharp rise (fall) in temperature is quite insignificant; in the boundary conditions, the behavior of temperature series no shorter than τ_{reg} determined for the given wall must be taken into account.

In practical multivariant calculations, when the time interval is tens and hundreds of hours, the use of explicit schemes with a step of 10^{-3}–10^{-4} h is uneconomical. High harmonics make a small contribution to the Fourier-series expansion of the extremal temperatures of the external air. Thus, in the expansion of 1940 minimum temperatures in Moscow over an interval

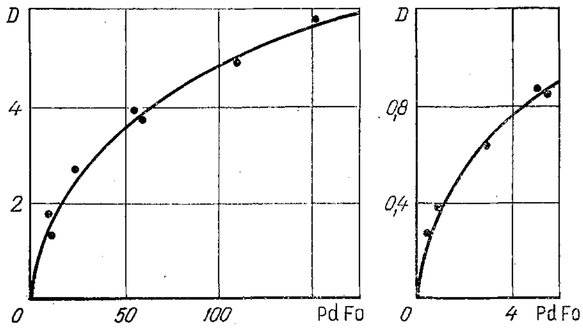

Fig. 1. Time of onset of regular heat transfer in external walls (generalization of the results of numerical calculations with $1.24 \leqslant Bi_E \leqslant 10$, $0.46 \leqslant Bi_I \leqslant 3.7$, $0.0524 \leqslant \omega \leqslant 0.524$, $\varepsilon = 10^{-5}$).

of 120 h, the amplitude of the harmonics (for the cosine expansion) with period T = 120 h is 13.42°C, while the amplitudes of harmonics with period $T \leqslant 20$ h is less than 0.4°C and, on passing through the wall, they make a contribution of ≈0.04°C to the temperature amplitude at the internal surface. Hence, in those cases where rapid calculation with a large step τ = 1-6 h is necessary, the use of the implicit scheme in Eq. (8) is expedient; in those cases where the high harmonics may distort the solution, it is necessary to decrease the step τ and to use the explicit scheme in Eq. (10). Thus, calculation of a wall with D = 1.21 by the implicit scheme with τ= 6 h gave minimum temperature 0.2°C higher than that calculated analytically from the accurate solution [4] for the minimum u_I in 1940 in Moscow.

NOTATION

x, coordinate; t, time; l, wall thickness; l_j, thickness of j-th layer of wall; γ, density; λ, thermal conductivity; c, specific heat; a, thermal diffusivity; α_E, heat-transfer coefficient of external surface of wall; α_I, heat-transfer coefficient of internal surface of wall; u, temperature; u_E, temperature of external air; u_c, mean temperature of external air; u_I, temperature of internal air; A_0, A, B, amplitudes; T, period; $\omega = 2\pi/T$, frequency; h_i, grid step along x axis; i, number of grid point along x axis; τ, grid step along t axis; y, grid function; N, number of layers in wall; τ_{reg}, time of onset of regular heat transfer at the internal surface of the wall; τ_m, maximum permissible step τ in explicit scheme; Δ, absolute error; $Pd = l^2\omega/a$, Predvoditelev number; $Fo = a\tau/l^2$, Fourier number; $Bi = \alpha/(\lambda c\gamma\omega)^{1/2}$, generalized Biot number.

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